CHAPTER 2

Section 2-1:

2-1. Let *a* and *b* denote a part above and below the specification, respectively.

$S = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

2-3. Let a denote an acceptable power supply. Let f, m, and c denote a power supply that has a functional, minor, or cosmetic error, respectively.

$S = \{a, f, m, c\}$

2-5. Let *y* and *n* denote a web site that contains and does not contain banner ads.
The sample space is the set of all possible sequences of *y* and *n* of length 24. An example outcome in the sample space is

$S\ \{yynnynyyynnynynnnnyynnyy\}$

2-7. A scale that displays two decimal places is used to measure material feeds in a chemical plant in tons.

S is the sample space of 100 possible two digit integers.

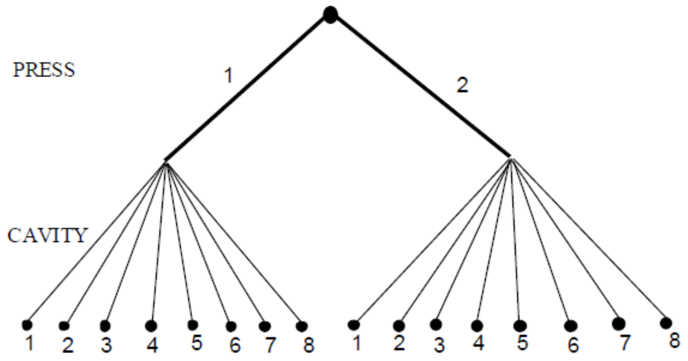
2-9. The concentration of ozone to the nearest part per billion.

S $\{0,1,2,...,1E09\}$ in ppb.

2-11. The pH reading of a water sample to the nearest tenth of a unit.
$S = \{1.0, 1.1, 1.2, \ldots 14.0\}$

2-13. The time of a chemical reaction is recorded to the nearest millisecond.
$S = \{0,1,2,...,\}$ in milliseconds.

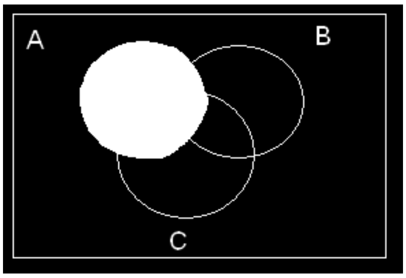
2-15.



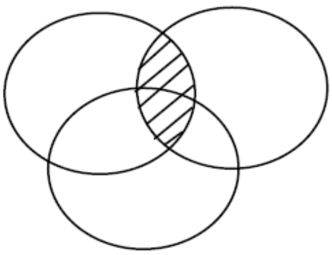
2-17. Let *c* and *b* denote connect and busy, respectively. Then *S* = {*c*, *bc*, *bbc*, *bbbc*, *bbbbc*, …}

2-19.

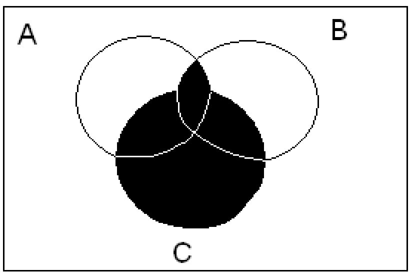
(a)



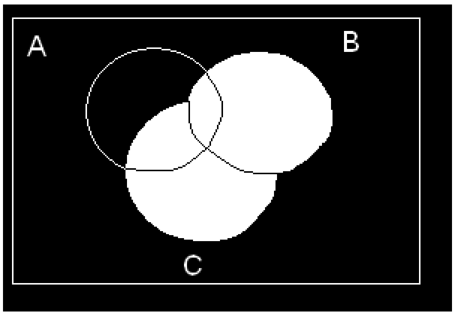
(b)



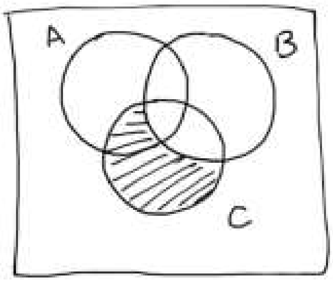
(c)



(d)



(e)



2-21.

(a) Let S = the nonnegative integers from 0 to the largest integer that can be displayed by the scale.

Let X denote the weight.

A is the event that $X > 11$ B is the event that $X \leq 15$ C is the event that $8 \leq X < 12$

$S = \{0, 1, 2, 3, \ldots\}$

(b) S

(c) $11 < X \leq 15$ or $\{12, 13, 14, 15\}$

(d) $X \leq 11$ or $\{0, 1, 2, \ldots, 11\}$

(e) S

(f) B' contains the values of X such that $X > 15$.

Thus $A \cap B'$ contains the values of X such that: $X > 15$ or $\{16, 17, 18, \ldots\}$

(g) $\varnothing$

(h) B' contains the values of X such that $X > 15$. Therefore, $B' \cap C$ is the empty set. They have no outcomes in common or $\varnothing$.

(i) $B \cap C$ is the event $8 \leq X < 12$. Therefore, $A \cup (B \cap C)$ is the event $X \geq 8$ or $\{8, 9, 10, \ldots\}$

2-23.

Let *d* and *o* denote a distorted bit and one that is not distorted (*o* denotes okay), respectively.

a) $$S=\left\{\begin{matrix} dddd,dddo,dddod,ddodd, \\ doddd,odddd,dddoo,ddodo, \\ doddo,odddo,ddood,dodod, \\ oddod,doodd,ododd,ooddd, \\ ddooo,dodoo,ooddo,doood, \\ ooodd,ododo,oddoo,odood, \\ doodo,doooo,odooo,oodoo, \\ ooodo,ooood,oodod,ooooo \end{matrix}\right\}$$

b) Ai's are not mutually exclusive.

$$A_1\cap A_2=\left\{\begin{matrix} ddddd,ddddo,dddod,ddodd, \\ dddoo,ddodo,ddood,ddooo \end{matrix}\right\}$$

c) $$A_1=\left\{\begin{matrix} ddddd,ddddo,dddod,ddodd, \\ doddd,dddoo,ddodo,doddo, \\ ddood,dodod,doodd,ddooo, \\ dodoo,doood,doodo,doooo \end{matrix}\right\}$$

d) $$A_1'=\left\{\begin{matrix} odddd\ odddo\ oddod\ ododd \\ ooddd\ ooddo\ ooodd\ ododo \\ oddoo\ odood\ odooo\ oodoo \\ ooodo\ ooood\ oodod\ ooooo \end{matrix}\right\}$$

e) $A_1\cap A_2\cap A_3\cap A_4=\{ddddd,ddddo\}$

f) $$(A_1\cap A_2)\cup(A_3\cap A_4)=\left\{\begin{matrix} ddddd,ddddo,dddod,ddodd,dddoo, \\ ddodo,ddood,ddooo,doddd,odddo, \\ odddd,doddo,ooddd,ooddo \end{matrix}\right\}$$

2-25.

Let *P* and *N* denote positive and negative, respectively.

The sample space is {*PPP*, *PPN*, *PNP*, *NPP*, *PNN*, *NPN*, *NNP*, *NNN*}.

(a) *A*={ *PPP* }

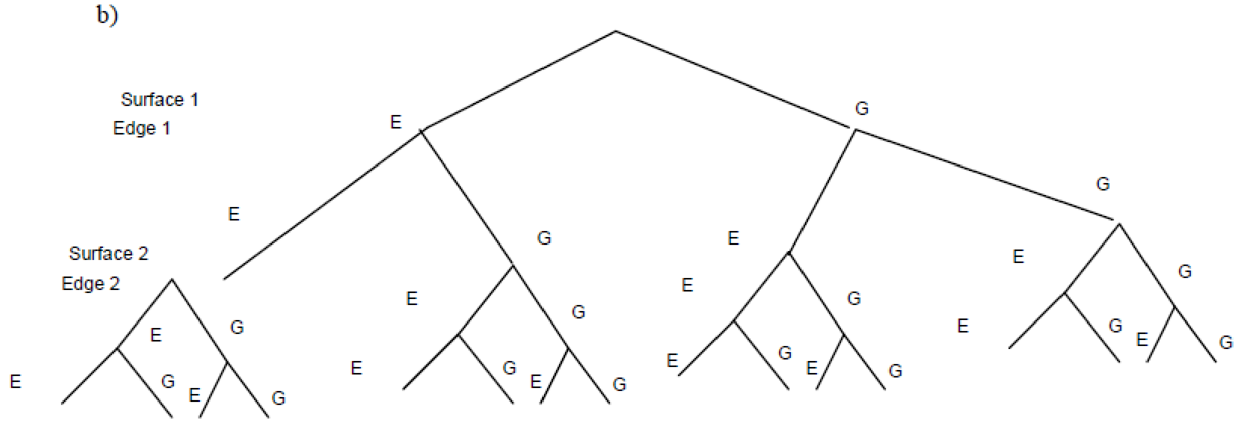
(b) B={ NNN }

(c) $A \cap B = \Phi$

(d) $(A \cup B)'$ = { PPN, PNP, NPP, PNN, NPN, NNP }

2-27. a) $A' \cap B = 10$, $B' = 10$, $A \cup B = 94$

b)



2-29.

(a) $A' = \{x \mid x \geq 65\}$

(b) $B' = \{x \mid x \leq 45.5\}$

(c) $A \cap B = \{x \mid 45.5 < x < 65\}$

(d) $A \cup B = \{x \mid x > 0\}$

2-31.

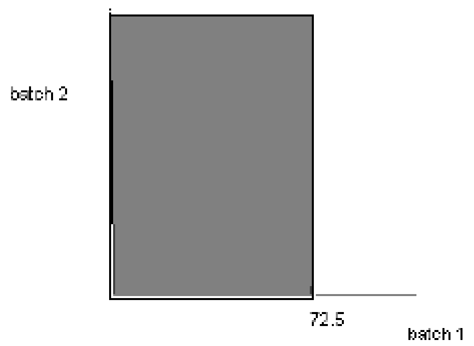
Let g denote a good board, m a board with minor defects, and j a board with major defects.

a) $S = \{gg, gm, gj, mg, mm, mj, jg, jm, jj\}$

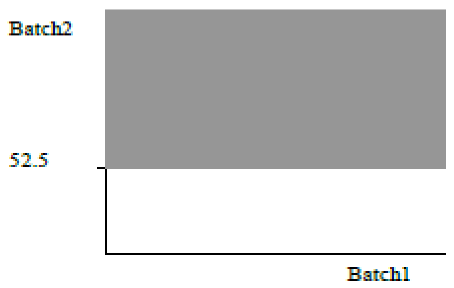
b) $S = \{gg,gm,gj,mg,mm,mj,jg,jm\}$

2-33.

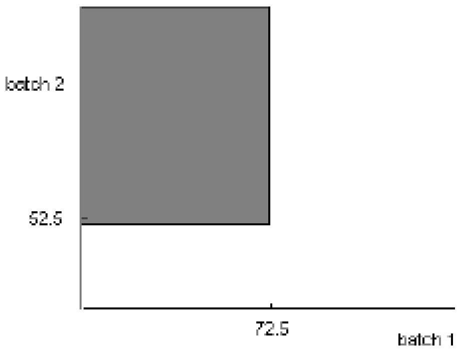
(a)


batch 2
72.5
batch 1

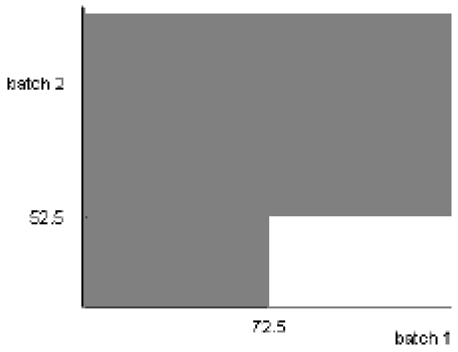
(b)


Batch2
52.5
Batch1

(c)


batch 2
52.5
72.5
batch 1

(d)


batch 2
52.5
72.5
batch 1

2-35. 150

2-37. 192

2-39. 576

2-41.

(a) 75,287,520
(b) 2,551,900
(c) 31,338,252

2-43.

(a) 28
(b) 20160
(c) 720

2-45.

(a) 1000
(b) 90
(c) 720

2-47.

(a) 0.454
(b) 0.597
(c) 0.166

2-49. 0.01282

2-51. 1125

2-53.

(a) 673
(b) 7726
(c) 6915
(d) 8399
(e) 1578

Section 2-2:

2-55.

a) 0.5 b) 0.7 c) 0.5 d) 1 e) 0.2

2-57.

(a) 1/10

(b) 6/10

2-59.

a) S = {1,2,3,4,5,6,7,8,9,10,11,12}

b) 2/12

c) 10/12

2-61.

(a) 0.85

(b) 0.85

2-63. 5.7×10^{-8}

2-65.

(a) 52

(b) 0.769.

(c) 0.769.

2-67.

(a) 0.30

(b) 0.75

(c) 0.70

(d) 0.2

(e) 0.85

(f) 0.9

2-69.

(a) Because E and E' are mutually exclusive events and $E \cup E' = S$

$1 = P(S) = P(E \cup E') = P(E) + P(E')$. Therefore, $P(E') = 1 - P(E)$

(b) Because S and $\emptyset$ are mutually exclusive events with $S = S \cup \emptyset$

$P(S) = P(S) + P(\emptyset)$. Therefore, $P(\emptyset) = 0$

(c) Now, $B = A \cup (A' \cap B)$ and the events A and $A' \cap B$ are mutually exclusive. Therefore,

$P(B) = P(A) + P(A' \cap B)$. Because $P(A' \cap B) \geq 0$, $P(B) \geq P(A)$.

2-71. 0.9911

2-73.

(a) 0.0903

(b) 0.1969

(c) 0.8142

(d) 0.9889

(e) 0.0111

Section 2-3:

2-75.

(a) 0.8

(b) 0

(c) 0

(d) 0

(e) 0.2

2-77.

(a) 0.70

(b) 0.95

(c) No, $P(A \cap B) \neq 0$

2-79.

(a) 349/370

(b) 361/370

(c) 358/370

(d) 334/370

2-81.

(a) 27/170

(b) 0.841, No

2-83.

(a) 0.9412

(b) 0.8039

(c) 0.7451

2-85.

(a) 0.2718

(b) 0.9659

(c) 0.9912

<u>Section 2-4:</u>

2-87.

(a) 85/100

(b) 80/100

(c) 7/8

(d) 7/8.5

2-89.

(a) 0.888

(b) 0.682

2-91.

(a) 36/150

(b) 36/52

(c) 34/148

2-93.

(a) 0.682

(b) 0.230

(c) 0.231

2-95.

a) 25/100

b) 24/99

c) 0.0606

d) 0.25

2-97.

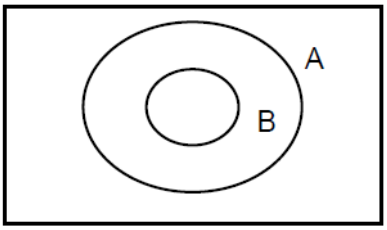
a) 0.023

b) 6.243×10^{-4}

c) 0.947

2-99.

No, if $B \subset A$, then $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$



2-101.

(a) 0.5

(b) 0.5

(c) 0.435

(d) 0.5

2-103.

(a) 0.0987

(b) 0.0650

Section 2-5:

2-105.

(a) 0.18

(b) 0.42

2-107. 0.024

2-109. 0.032

2-111.

(a) 0.212

(b) 0.126

2-113.

(a) 0.20

(b) 0.20

2-115.

Open surgery

	success	failure	sample size	sample percentage	conditional success rate
large stone	192	71	263	75%	73%
small stone	81	6	87	25%	93%
overall summary	273	77	350	100%	78%

PN

	success	failure	sample size	sample percentage	conditional success rate
large stone	55	25	80	23%	69%
small stone	234	36	270	77%	83%
overall summary	289	61	350	100%	83%

The overall success rate depends on the success rates for each stone size group, but also the probability of the groups. It is the weighted average of the group success rate weighted by the group size as follows

P(overall success) = P(success| large stone)P(large stone)) + P(success| small stone)P(small stone).

For open surgery, the dominant group (large stone) has a smaller success rate while for PN, the dominant group (small stone) has a larger success rate.

2-117.

(a) 0.0088

(b) 0.2718

(c) 0.9912

(d) 0.2378

2-119.

(a) 0.0792

(b) 0.8142

(c) 0.9208

(d) 0.8031

2-121. 0.222

Section 2-6:

2-123. A' and B are not independent events.

2-125.

(a) *A* and *B* are not independent.

(b) *A* and *B* are not independent.

2-127.

(a) A and B are not idependent.

(b) 0.733

2-129.

(a) 0.6561

(b) 0.2916

(c) 0.3439

2-131.

(a) 0.0111

(b) 0.0908

2-133.

(a) 0.01

(b) 0.49

(c) 0.09

2-135.

(a) 0.00003

(b) 0.00024

(c) 0.00107

2-137. 0.9483

2-139. A and B are not independent.

2-141. A and B are independent.

<u>Section 2-7:</u>

2-143. 0.921

2-145.

(a) 0.97638

(b) 0.207552

2-147.

(a) 0.6325

(b) 0.6759

(c) 0.0612

2-149.

(a) 0.9847

(b) 0.1184

2-151. 0.2540

2-153. 0.5

Section 2-8:

2-155.

(a) discrete (b) continuous (c) discrete, but large values might be modeled as continuous

(d) continuous (e) continuous

2-157. 0.024

2-159.

a) 0.73

b) 0.90

c) 0.27

d) 0.70

e) 0.93

f) 0.97

2-161. P(A), P(B),and P(C) cannot equal the given values.

2-163.

(a) 0.1

(b) 0.101

(c) 0.809

(d) 0.1.

The event of the first selection and the event of the second selection are independent.

2-165.

a) 0.02

b) 0.98

c) 0.5

d) 0.04

e) 0.01

f) 0.01

g) 0.0492

2-167.

(a) 0.3942
(b) 0.00149

(c) 0.9933

2-169. 0.000064

2-171.

(a) 42

(b) 45

(c) 93

2-173. S = {A,A'D1,A'D2,A'D3,A'D4,A'D5} .

2-175.

a) 0.24

b) 0.1667

c) 0.95

d) 0.7583

e) 0.146

2-177.

a) No, $P(E1 \cap E2 \cap E3) \neq 0$

b) No, $E1' \cap E2'$ is not $\varnothing$

c) $P(E1' \cup E2' \cup E3') = P(E1') + P(E2') + P(E3') – P(E1'\cap E2') - P(E1'\cap E3') - P(E2'\cap E3') + P(E1' \cap E2' \cap E3')$

$= 40/240$

d) $P(E1 \cap E2 \cap E3) = 200/240$

e) $P(E1 \cup E3) = P(E1) + P(E3) – P(E1 \cap E3) = 234/240$

f) $P(E1 \cup E2 \cup E3) = 1 – P(E1' \cap E2' \cap E3') = 1 - 0 = 1$

2-179.

(a) 0.496

(b) 0.504

2-181. 0.995

2-183.

(a) 0.0086

(b) 0.9302

2-185.

(a) 0.049

(b) 0.00095

(c) 0.95

2-187.

(a) 0.9639

(b) 0.2888

2-189.

(a) 0.207

(b) 0.625

(c) 0.60

2-191.

(a) 0.453

(b) 0.27

(c) 0.881

(d) 0.547

(e) 0.809

(f) 0.694

2-193. $4.75*10^{-6}$

2-195.

(a) 0.67336

(b) 2.646× 10-8

(c) 0.99973

2-197.

(a) 36^7

(b) Number of permutations of six letters is 26^6. Number of ways to select one number = 10. Number of positions

among the six letters to place the one number = 7. Number of passwords = $26^6 \times 10 \times 7$

(c) $26^5 10^2$

2-199.

(a)

For supplier 1: 0.994

For supplier 2: 0.995

(b)

For supplier 1: 0.99

For supplier 2: 0.985

(c)

For supplier 10.998

For supplier 2: 0.9975

(d) The unusual result is that for both a simple component and for a complex assembly, supplier 1 has a greater probability that a part conforms to specifications. However, supplier 1 has a lower probability of conformance overall. The overall conforming probability depends on both the conforming probability of each part type and also the probability of each part type. Supplier 1 produces more of the complex parts so that overall conformance from supplier

1 is lower.

Mind-Expanding Exercises:

2-201.

(a) 4

(b) 4

2-203. 0.694

2-205.

Suppose that a table of part counts is generalized as follows:

		Conforms	
		Yes	No
Supplier	1	ka	kb
	2	a	b

where a, b, and k are positive integers. Let A denote the event that a part is from supplier 1, and let B denote the event that a part conforms to specifications. Show that A and B are independent events. This exercise illustrates the result that whenever the rows of a table (with r rows and c columns) are proportional, an event defined by a row category and an event defined by a column category are independent.

The total sample size is $ka + a + kb + b = (k+1)a + (k+1)b$. Therefore

$$P(A) = \frac{k(a+b)}{(k+1)a+(k+1)b}, P(B) = \frac{ka+a}{(k+1)a+(k+1)b}$$

and

$$P(A \cap B) = \frac{ka}{(k+1)a+(k+1)b} = \frac{ka}{(k+1)(a+b)}$$

Then,

$$P(A)P(B) = \frac{k(a+b)(ka+a)}{[(k+1)a+(k+1)b]^2} = \frac{k(a+b)(k+1)a}{(k+1)^2(a+b)^2} = \frac{ka}{(k+1)(a+b)} = P(A \cap B)$$