STUDY SET 5

Hypothesis Tests of a Single Population

1. Differentiate between a one-sided and two-sided composite alternative hypothesis when the null hypothesis is $H_0 : \mu = 10$?

ANSWER:
When the null hypothesis is $H_0 : \mu = 10$, a possible alternative hypothesis would be $H_1 : \mu > 10$. This alternative hypothesis suggests that population mean value falls in a range of values greater than 10, the value specified in the null hypothesis. Hence, we define this alternative hypothesis as a one-sided composite. Another possibility would be to test the null hypothesis against the general two-sided composite alternative hypothesis: $H_1 : \mu \neq 10$.

This hypothesis suggests that the value could be lesser or greater than the value specified in the null hypothesis. We choose these hypotheses so that one or the other must be true.

2. A music industry professional claims that the average amount of money that an average teenager spends per month on music is at least $50. Based upon previous research, the population standard deviation is estimated to be $12.42. The music professional surveys 20 students and finds that the mean spending is $47.77. Is there evidence that the average amount spent by students is less than $50?

ANSWER:
There is no sufficient evidence to disprove the claim that the average amount spent by students is at least $50.

3. Give an account of how a decision is made concerning the null hypothesis.

ANSWER:
Once we have specified the null and the alternative hypotheses and collected the sample data, we must make a decision concerning the null hypothesis. We can either reject the null hypothesis and accept the alternative, or fail to reject the null hypothesis. For good reasons many statisticians prefer not to say, “accept the null hypothesis”; instead, they say, “fail to reject the null hypothesis.” When we fail to reject the null hypothesis, then either the null hypothesis is true or our test procedure was not strong enough to reject it and we have committed an error. To select the hypothesis—null or alternative—we develop a decision rule based on sample evidence.

4. Explain carefully the distinction between simple and composite hypotheses.

ANSWER:
A simple hypothesis assumes a specific value for the population parameter that is being tested. A composite hypothesis assumes a range of values for the population parameter.

5. Explain the relationship between Type I error and Type II error in a hypothesis test.

ANSWER:
Ideally, we would like to have the probabilities of both types of errors be as small as possible. However, there is a trade-off between the probabilities of the two types of errors. Given a particular sample, any reduction in the probability of Type I error, α, will result in an increase in the probability of Type II error, β, and vice versa, although it is not a direct linear substitution.

6. The supervisor of a production line believes that the average time to assemble an electronic component is 14 minutes. Assume that assembly time is normally distributed with a standard deviation of 3.4 minutes. The supervisor times the assembly of 14 components, and finds that the average time for completion was 11.6 minutes. Is there evidence that the average amount of time required to assemble a component is something other than 14 minutes? Use $\alpha = 0.01$.

ANSWER:
We conclude that there is sufficient evidence to disprove the claim that the average time to assemble an electronic component is 14 minutes (The population mean appears to be less than 14 minutes.)

7. Explain the distinction between Type I and Type II errors.

ANSWER:
A Type I error is falsely rejecting the null hypothesis. To commit a Type I error, the truth must be that the null hypothesis is really true and yet you conclude to reject the null and accept the alternative. A Type II error is falsely not rejecting the null hypothesis when in fact the null hypothesis is false. To make a Type II error, the null hypothesis must be false (the alternative is true) and yet you conclude not to reject the null hypothesis.

8. How is the power of a test connected to the decision rule in hypothesis testing?

ANSWER:
The decision rule is determined by the significance level chosen for the test, therefore the concept of power does not directly affect the decision to reject or fail to reject a null hypothesis. However, by computing the power of the test for particular significance levels and values of population parameters included in H_1, we will have valuable information about the properties of the decision rule. For

example, we will see that, by taking a larger sample size, the power of the test will be increased for a given significance level, α. Another important use of power calculations occurs when, for a given sample size, we have a choice between two or more possible tests with the same significance levels. Then it would be appropriate to choose the test that has the smallest probability of Type II error—that is, the test with the highest power.

9. Explain the distinction between the significance level and the power of the test.

ANSWER:
Significance level is the chosen level of significance that established the probability of a making a Type I error. This is represented by α. A Type II error, that arises when we fail to reject a false null hypothesis. For a particular decision rule, the probability of making such an error when the null hypothesis is false will be denoted as β. Then the probability of rejecting a false null hypothesis is $(1-\beta)$, which is called the power of the test.

10. The manufacturer of a new product claims that his product will increase output per machine by at least 38 units per hour. A line manager adopts the product on 18 of his machines, and finds that the average increase was only 35 units per hour with a standard deviation of 12.2 units. Is there evidence to doubt the manufacturer's claim at a significance level of .10?

ANSWER:
There is insufficient evidence to disprove the claim that the product will increase output per machine by 38 units per hour.

11. How does sample size affect the decision to reject or failure to reject of a null hypothesis?

ANSWER:
The null hypothesis has the status of a maintained hypothesis—one held to be true—unless the data contain strong evidence to reject the hypothesis. By setting the significance level, α at a low level, we have a small probability of rejecting a true null hypothesis. When we reject a true null hypothesis, the probability of error is the significance level, α. But if there is only a small sample, then we will reject the null hypothesis only when it is wildly in error. As we increase the sample size, the probability of rejecting a false null hypothesis is increased. But failure to reject a null hypothesis leads to much greater uncertainty because we do not know the probability of the Type II error. Thus, if we fail to reject, then either the null hypothesis is true or our procedure for detecting a false null hypothesis does not have sufficient power—for example, the sample size is too small.

12. The manufacturer of a new chewing gum claims that at least eight out of ten dentists surveyed prefer their type of gum and recommend it for their patients who chew gum. An independent consumer research firm decides to test their

claim. The findings in a sample of 400 doctors indicate that 76% do actually prefer their gum. Is this evidence sufficient to doubt the manufacturer's claim? Use $\alpha = 0.05$.

ANSWER:
There is sufficient evidence to doubt the manufacturer's claim.

13. A magazine advertises that at least 65% of all their readers are women. A woman's group wants to test this assertion and they take a sample of 300 households, and find that only 61.3% of the readers are women. From the data, is there sufficient evidence to doubt the magazine's claim at a 10% level of significance?

ANSWER:
Therefore, there is sufficient evidence to doubt the magazine's claim at the 10% level of significance.

14. How is hypothesis testing a counterfactual argument?

ANSWER:
When we reject the null hypothesis, we have strong evidence that the null hypothesis is not true and, therefore, that the alternative hypothesis is true. If we seek strong evidence in favor of a particular outcome, we define that outcome as the alternative hypothesis, H_1, and the other outcome as the null hypothesis, H_0. This is called a counterfactual argument. When we reject H_0, there is strong evidence in favor of H_1, and we are confident that our decision is correct. But failing to reject H_0 leads to greater uncertainty.

15. The manufacturer of bags of cement claims that they fill each bag with at least 50.2 pounds of cement. Assume that the standard deviation for the amount in each bag is 1.2 pounds. The decision rule is adopted to shut down the filling machine if the sample mean weight for a sample of 40 bags is below 49.8. Suppose that the true mean weight of bags is 50 pounds. Using this decision rule, what is the probability of a Type II error?

ANSWER:
0.1469

16. A professional rock climber claims that he can climb a particular mountain within half an hour. From a random sample of 20 attempts, the rock climber averaged 33.3 minutes with a standard deviation of 3.9 minutes. Is there sufficient evidence to suggest that the climber's claim is incorrect at a significance level of 10 per cent?

ANSWER:

There is sufficient evidence to disprove the climber's claim.

17. Thomas is a shift manager at a local fast food place, and is responsible for quality management. Thomas wants to ensure that all the frozen hamburger patties that get delivered by the supplier weigh four ounces on average. Assume that the standard deviation of the weight of hamburger patties is known to be 0.1 ounces. Thomas tells one of his employees that as a shipment arrives, select 25 patties at random and find the average weight for the 25 patties. For what average weight would you tell the employee to reject the shipment, if you wanted the probability of a Type I error to be 0.10 or less?

    ANSWER:
    Reject the shipment if the average weight of the 250 sampled patties is <3.9744 ounces.

18. Provide an example of two-sided alternative hypotheses.

    ANSWER:
    There are some problems where deviations either too high or too low are of equal importance. Such problems are tested using a two-sided alternative hypothesis. For example, the diameter of an automobile engine piston cannot be too large or too small. In those situations we consider the test of the null hypothesis

$$H_0 : \mu = \mu_0$$

    against the alternative hypothesis

$$H_1 : \mu \neq \mu_0$$

    Here, we have no strong reason for suspecting departures either above or below the hypothesized population mean, μ_0. The null hypothesis would be doubted if the sample mean were much greater or much smaller than μ_0.

19. An assembly line will be shut down for maintenance if the defect rate exceeds 4.6%. Suppose you adopt a 5% significance level and take a random sample of 375 items off the assembly line and compute the proportions that are defective. For what value of the sample proportion will the assembly line be shut down?

    ANSWER:
    Assembly line will be shut down for maintenance if ($\hat{p}$ -0.046) / 0.0108 > 1.645, or equivalently if the sample proportion exceeds 0.0637.

20. A gaming control board claims that the average household income of those people playing blackjack at casinos is at least $50,000. Assume that the distribution of household income of those people playing blackjack is normally distributed with a standard deviation of $7,559. Suppose that for a sample of 45 households, it is found that the average income was $48,326. Is the gaming control board incorrect

in asserting that the average household income of blackjack players is at least \$50,000? Use the 10 per cent significance level.

ANSWER:
There is sufficient evidence to disprove the claim of the gaming control board.

21. A fitness club claims that the average age of its members is 42. Assume that the ages are normally distributed. You believe that the average age is less than 42, therefore, you ask the ages of a few of the fitness club's members and get the following values: 40, 55, 50, 33, 38, 35, 30, and 45. Do you have any reason at the 10 per cent level of significance to think the fitness club's claim is incorrect?

ANSWER:
Therefore, there is insufficient evidence to disprove the claim.

22. List out the features of a power function in hypothesis testing.

ANSWER:
The power function has the following features:
1. The farther the true mean is from the hypothesized mean μ_0, the greater is the power of the test—everything else being equal.
2. The smaller the significance level(α) of the test, the smaller the power—everything else being equal. Thus, reducing the probability of Type I error α increases the probability of Type II error β, but reducing α by 0.01 does not generally increase β by 0.01; the changes are not linear.
3. The larger the population variance, the lower the power of the test—everything else being equal.
4. The larger the sample size, the greater the power of the test—everything else being equal. Note that larger sample sizes reduce the variance of the sample mean and, thus, provide a greater chance that we will reject H_0 when it is not correct.
5. The power of the test at the critical value is the probability that a sample mean is above ($\bar{x}_c$), is that value.

23. A local transportation planning group is concerned about the lack of car-pooling on the part of commuters. They are afraid that the proportion of local drivers car-pooling is below the national average of 15%. A survey of 400 local drivers reveals that 12.22% of them car pool. Is there evidence that the actual proportion of local commuters car-pooling is less than 15%? Test at the 10% level of significance.

ANSWER:
There is sufficient evidence that the actual proportion of local commuters car-pooling is less than 15%.

24. Explain the hypothesis test of the population proportion for large sample sizes.

ANSWER:
We begin by assuming a random sample of *n* observations from a population that has a proportion *P* whose members possess a particular attribute. If $nP(1 - P) > 5$ and the sample proportion is $\hat{p}$, then the following tests have significance level α.

1. To test either the hypothesis

$$H_0 : P = P_0 \text{ or } H_1 : P \leq P_0$$

against the alternative

$$H_1 : P > P_0$$

the decision rule is as follows:

$$\text{reject } H_0 \text{ if } \frac{\hat{p} - P_0}{\sqrt{P_0(1 - P_0)/n}} > z_{\alpha}$$

2. To test either null hypothesis

$$H_0 : P = P_0 \text{ or } H_1 : P \geq P_0$$

against the alternative

$$H_1 : P < P_0$$

the decision rule is as follows:

$$\text{reject } H_0 \text{ if } \frac{\hat{p} - P_0}{\sqrt{P_0(1 - P_0)/n}} < -z_{\alpha}$$

3. To test either null hypothesis

$$H_0 : P = P_0$$

against the alternative

$$H_1 : P \neq P_0$$

the decision rule is as follows:

$$\text{reject } H_0 \text{ if } \frac{\hat{p} - P_0}{\sqrt{P_0(1 - P_0)/n}} < -z_{\alpha/2}$$

$$\text{or reject } H_0 \text{ if } \frac{\hat{p} - P_0}{\sqrt{P_0(1 - P_0)/n}} > z_{\alpha/2}$$

25. Suppose that you want to test $H_0 : \mu = 277$ against $H_1 : \mu \neq 277$ at $\alpha = 0.10$, and you know that the population standard deviation $\sigma = 13.5$. If you randomly select a sample of 20 observations, for what values of the sample mean will you reject the null hypothesis?

ANSWER:
Reject H_0 for sample mean $\bar{x} = k < 277 - 1.645(3.019) = 272.03$ or sample mean $\bar{x} = k > 277 + 1.645(3.019) = 281.97$.

26. Provide the test for mean of a normal distribution with population variance unknown, with the null hypothesis $H_0 : \mu \le \mu_0$.

ANSWER:
Since the null hypothesis is

$$H_0 : \mu \le \mu_0$$

the alternate hypothesis would be

$$H_0 : \mu > \mu_0$$

the decision rule is

$$\text{reject } H_0 \text{ if } t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} > t_{n-1,\alpha}$$

or equivalent

$$\text{reject } H_0 \text{ if } \bar{x} > \bar{x}_c = \mu_0 + t_{n-1,\alpha}\, s / \sqrt{n}$$

<u>THE NEXT TWO QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:</u>
The marketing department for a hand calculator claims the battery pack performs 20,000 calculations before it has to be recharged. The quality control manager for the manufacturer is entrusted with validating the claim that the battery pack works as long as the specifications state in order to secure a large order from a key customer. A test of 114 battery packs yields an average of 19,695 calculations with a standard deviation of 1,103.

27. Formulate the appropriate null and alternative hypotheses.

ANSWER:
$H_0:\ \mu \ge 20{,}000,\ \ H_1:\ \mu < 20{,}000.$

28. Calculate the appropriate test statistic and *p*-value.

ANSWER:
t = -2.952
Since *df* = *n* -1 = 113, and $t_{\infty,0.005} = 2.576$, then *p*-value < 0.005.

29. How is the *p*-value computed for a two-sided alternative given a null hypothesis $H_0 : \mu = \mu_0$ for the mean of a normal distribution with known variance?

ANSWER:
The *p*-values can be computed by noting that the corresponding tail probability would be doubled to reflect a *p*-value that refers to the sum of the upper- and lower-tail probabilities for the positive and negative values of *z*. the *p*-value for the two-tailed test of $H_0 : \mu = \mu_0$ is

p-value = $2P\left(\left|\frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}\right| > z_{p/2} \middle| H_0 : \mu = \mu_0\right)$

where $z_{p/2}$ is the standard normal value associated with the smallest probability of rejecting the null hypothesis at either tail of the probability distribution.

<u>THE NEXT THREE QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:</u>
A team of builders has surveyed buyers of their new homes for years. Consistently, only 41% of the buyers have indicated that they were "quite satisfied" or "very satisfied" with the construction quality of their homes. The builders have implemented new procedures and adopted a revised quality-inspection system to ultimately improve customer satisfaction. They have surveyed 104 buyers since implementing changes; these buyers seem representative, with no systematic changes from past purchasers. Of the 104 buyers, 52 indicated they were quite or very satisfied.

30. What kind of alternative hypothesis type would fit such a survey?

ANSWER:
$H_1 :\ P > 0.41$

31. Calculate the appropriate statistic for testing the null hypothesis.

ANSWER:
1.87

32. Test the null hypothesis with the alternative hypothesis, $H_1 :\ P > 0.41$, at $\alpha = 0.05$.

ANSWER:

We conclude there is sufficient evidence that adopting the revised quality-inspection system has improved the proportion of customer satisfaction.

33. How is the probability β making a Type II error found in population proportion tests?

ANSWER:
The probability, β, of making a Type II error for any given population proportion P_1 included in H_1 is found as follows:
1. From the test decision rule, find the range of values of the sample proportion leading to failure to reject the null hypothesis.
2. Using the value P_1 for the population proportion—where P_1 is included in the alternative hypothesis—find the probability that the sample proportion will be in the nonrejection region determined in step 1 for samples of n observations when the population proportion is P_1.

THE NEXT THREE QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:
A pharmaceutical manufacturer is concerned that the mean impurity concentration in pills should not exceed 3%. It is known that from a particular production run impurity concentrations follow a normal distribution with a standard deviation 0.4%. A random sample of 36 pills from a production run was checked, and the sample mean impurity concentration was found to be 3.09%.

34. Test at the 5% level, the null hypothesis that the population mean impurity concentration is 2% or less against the alternative that it is more than 3%.

ANSWER:
We conclude that the population mean impurity concentration is 3% or less.

35. Calculate the *p*-value for this test.

ANSWER:
p -value = $P($ $Z > 1.35 = 0.0885$

36. In the context of this problem, explain why a one-sided alternative hypothesis is more appropriate than a two-sided alternative.

ANSWER:
A one-sided alternative is more appropriate since we are not interested in detecting possible low levels of impurity, only high levels of impurity.

37. How is hypothesis testing used in problems that have high variance levels?

ANSWER:
Here, we will develop procedures for testing the population variance, σ^2, based

on the sample variance, s^2, computed using a random sample of n observations from a normally distributed population. If the null hypothesis is that the population variance is equal to some specified value, that is,

$$H_0 : \sigma^2 = \sigma^2{}_0$$

then when this hypothesis is true, the random variable

$$X^2_{n-1} = \frac{(n-1)s^2}{\sigma_0^2}$$

has a chi-square distribution with (n - 1) degrees of freedom. Hypothesis tests are based on computed values of this statistic. If the alternative hypothesis were

$$H_1 : \sigma^2 > \sigma_0^2$$

we would reject the null hypothesis if the sample variance greatly exceeded σ_0^2. Thus, a high computed value of X^2_{n-1} would result in the rejection of the null hypothesis. Conversely, if the alternative hypothesis were

$$H_1 : \sigma^2 < \sigma_0^2$$

we would reject the null hypothesis if the value of X^2_{n-1} were small. For a two-sided alternative

$$H_1 : \sigma^2 \neq \sigma_0^2$$

we would reject the null hypothesis if the computed X^2_{n-1} were either unusually high or unusually low.

<u>THE NEXT THREE QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:</u>
An engineering research center claims that, through the use of a new computer control system, automobiles should achieve, on average, at least an additional 5 miles per gallon of gas. A random sample of 81 automobiles was used to evaluate this product. The sample mean increase in miles per gallon achieved was 4.5, and the sample standard deviation was 2.9 miles per gallon.

38. State the appropriate null and alternative hypotheses.

ANSWER:
$H_0 : \mu \geq 5$ and $H_1 : \mu < 5$.

39. Test the hypothesis at the 5% level.

ANSWER:
We conclude that through the use of a new computer control system, automobiles should achieve, on average, at least an additional 5 miles per gallon of gas.

40. Find the p-value of this test, and interpret the results.

ANSWER:

p-value = $P(t < -1.55) = 0.0623$. This means that H_0 would be rejected for any significance level above 6.23%.

41. When is the chi-square distribution test not reliable for testing in hypothesis tests?

ANSWER:
The chi-square distribution tests are more sensitive to the assumption of normality in the underlying distribution compared to the standard normal distribution tests. Thus, if the underlying population deviates considerably from the normal, the significance levels computed using the chi-square distribution and the hypothesis tests may not be correct.

THE NEXT THREE QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:
A company selling new econometrics computer software advertises that firms using this software obtain, on average during the first year, a yield of at least 10% on their initial investments. A random sample of 10 of the franchises that used the software produced the following yields for the first year of operation: 9.4, 11.1, 10.4, 10.5, 11.2, 8.2, 8.5, 4.0, 8.8, and 6.1. Assume that population yields are normally distributed.

42. State the appropriate null and alternative hypotheses.

ANSWER:
$H_0 : \mu \geq 10$ and $H_1 : \mu < 10$.

43. Calculate the sample mean and sample standard deviation.

ANSWER:
$\bar{x} = 8.82$ and $s = 2.301$

44. Test the company's claim at the 5% significance level.

ANSWER:
We conclude that firms using this new software obtain, on average during the first year, a yield of at least 10% on their initial investments.

45. How does variance affect hypothesis testing in most applied situations?

ANSWER:
In most applied situations, and especially in quality-control work, the concern is about variances that are larger than anticipated. A variance that is smaller than anticipated results in hypothesis tests with greater power and confidence intervals that are narrower than anticipated. The opposite is true when the variance is larger than anticipated. Therefore, in most applied situations we are interested in the alternative hypothesis

$$H_1 : \sigma^2 > \sigma_0^2$$

where the variance is larger than expected.

46. On the basis of a random sample the null hypothesis: $H_0 : \mu \leq \mu_0$ is tested against the alternative: $H_1 : \mu > \mu_0$ and the null hypothesis is not rejected at the 10% significance level. Does this necessarily imply that μ_0 is contained in the 90% confidence interval for μ?

ANSWER:
No, the 90% confidence level provides for 5% of the area in either tail. This does not correspond to a one-tailed hypothesis test with α = 10% which has 10% of the area in one of the tails.

THE NEXT TWO QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:
In a random sample of 200 high school students, 92 sample members indicated some measure of agreement with this statement: "A prospective university's academic ranking and image is important to my decision to attend that institution".

47. Test at the 5% level of significance the null hypothesis that one-half of all high school students would agree with this statement against a two-sided alternative.

ANSWER:
We conclude that the proportion of high school students who would agree with the above statement is not different from 0.50.

48. Find and interpret the *p*-value of the test.

ANSWER:
The probability of finding a random sample with a sample proportion $\hat{p}$ this far or farther from 0.5 if the null hypothesis is really true is 0.2584.

49. The Discovery Automobile Company is attempting to determine if it should retain a previously popular sports car model called El Nino. A random sample of 500 consumers is obtained, and each person in the sample is asked if he/she would still consider purchasing an El Nino. To determine if El Nino should be retained, the sample proportion of respondents who said yes, $\hat{p}$, is used at a level α = 0.05 to test the hypothesis $H_0 : P \geq 0.5$ against $H_1 : P < 0.5$. What value of the sample proportion, $\hat{p}$, is required to reject the null hypothesis?

ANSWER:
$\hat{p} = 0.4636$ or 46.36%.

50. A random sample of 250 business faculty members was asked if there should be a required foreign language course for international business majors. Of these sample members, 170 felt there was a need for a foreign language course. Test the

hypothesis that at least 75% of all business faculty members hold this view. Use $\alpha = 0.05$.

ANSWER:
We conclude that less than 75% of all business faculty members hold the view that there should be a required foreign language course for international business majors.

THE NEXT FOUR QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:
Consider a problem with the hypothesis test $H_0: \mu = 4$ against $H_1: \mu > 4$, and the following decision rule: Reject H_0 if $\frac{\bar{x} - 4}{0.08/\sqrt{25}} > 1.645$.

51. Rewrite the decision rule explicitly in terms of $\bar{x}$.

ANSWER:
$\frac{\bar{x} - 4}{0.08/\sqrt{25}} > 1.645 \Rightarrow \bar{x} > 4 + 1.645\left(0.08/\sqrt{25}\right) = 4.026 \Rightarrow$ Reject H_0 if $\bar{x} > 4.026$

52. Compute the probability of Type II error and the power of the test for $\mu = 4.06$.

ANSWER:
Power = 1 - β = 1 – 0.0166 = 0.9834

53. Compute the probability of Type II error and the power of the test for $\mu = 4.04$.

ANSWER:
Power = 1 - β = 1 – 0.1894 = 0.8106

54. Compute the probability of Type II error and the power of the test for $\mu = 4.0$.

ANSWER:
Power = 1 - β = 1 – .9484 = 0.0516.

THE NEXT TWO QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:
Manufacturing planning wants to estimate plant capacity in order to schedule customer orders and improve on-time delivery. A random sample of ten days output gave the following results for numbers of finished components produced: 577, 642, 615, 655, 600, 570, 640, 620, 595, and 575. Planning is concerned about the variability of daily output and views as undesirable any variance above 525.

55. Calculate the sample variance.

ANSWER:
$s^2 = 922.322$

56. Test at the 10% significance level the null hypothesis that the population variance for daily output does not exceed 525.

ANSWER:
We conclude that the population variance for daily output exceeds 525.

57. A large milling machine produces steel rods. The machine is considered to be operating within customer specifications if the standard deviation of the diameter of the rods is 0.15 inches. As line supervisor, you need to demonstrate whether the machine is capable of meeting customer specifications. Based on a sample of 25 rods, the sample standard deviation is 0.19. Is there sufficient evidence at the 5% significance level to conclude that the machine is not operating within the customer tolerances?

ANSWER:
There is sufficient evidence at the 5% significance level to conclude that the machine is not operating normally.

58. The concentration of a drying compound is critical in gauging the final strength of an adhesive product. The standard deviation of the amount of the compound must be 1,500 parts-per-million (ppm) or less. Based on a sample of 25 mixtures, the standard deviation is 1,700 ppm. Is there reason to believe, at the 10 per cent significance level, that the standard deviation of the drying compound is more than 1,500 ppm? (Note: 1,500 ppm equals a proportion of .0015 and a percent of .000015.)

ANSWER:
We fail to reject null hypothesis at $\alpha = 0.10$.
There is no reason to believe at $\alpha = 0.10$ that the standard deviation of the drying compound is more than 1,500 parts-per-million.

<u>THE NEXT THREE QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:</u>
A beer producer claims that the proportion of its customers who cannot distinguish regular from light beer is at most 12 percent. The producer decides to test this null hypothesis against the alternative that the true proportion is more than 12 percent. The decision rule adopted is to reject the null hypothesis if the sample proportion of who cannot distinguish between the two beers exceeds 15 percent.

59. If a random sample of 100 customers is chosen, what is the probability of a Type I error, using this decision rule?

ANSWER:

$H_0 : P \leq 0.12$ and $H_1 : P > 0.12$.
The decision rule is: Reject H_0 if $\hat{p} > 0.15$
0.1788

60. If a random sample of 400 customers is selected, what is the probability of a Type I error, using this decision rule?

ANSWER:
0.0322

61. Suppose that the true proportion of customers who cannot distinguish between these flavors is 0.10. If a random sample of 100 customers is selected, what is the probability of a Type II error?

ANSWER:
0.1056

THE NEXT THREE QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:
An insurance company employs agents on a commission basis. It claims that in their first year, agents will earn a mean commission of at least \$42,000 and that the population standard deviation is no more than \$6,800. A random sample of nine agents found, for commissions in the first year, $\sum_{i=1}^{9} x_i = 351,000$ and $\sum_{i=1}^{9} (x_i - \bar{x})^2 = 338,000,000$.
The population distribution can be assumed to be normal.

62. What are the appropriate null and alternative hypotheses?

ANSWER:
$H_0 : \mu \geq 42,000$ and $H_1 : \mu < 42,000$

63. Calculate the sample mean and sample standard deviation.

ANSWER:
$\bar{x} = \sum_{i=1}^{9} x_i / 9 = 351,000/9 = 39,000$
and
$s = \sqrt{\sum_{i=1}^{9} (x_i - \bar{x})^2 / 8} = \sqrt{338,000,000/8} = 6,500$.

64. Test at the 5% level the null hypothesis $H_0 : \mu \geq 42,000$ against the alternative hypothesis $H_1 : \mu < 42,000$.

ANSWER:
We conclude that insurance company agents, in their first year, earn a mean commission of at least \$42,000.

<u>THE NEXT TWO QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:</u>
Research and development has designed a ball point pen so that the true average writing lifetime under controlled conditions (involving the use of a writing machine) is at least 12 hours. A random sample of 18 pens is selected, the writing lifetime of each is determined, and a normal probability plot of the resulting data supports the use of a one-sample *t* test.

65. What conclusion is appropriate for the hypotheses $H_0: \mu = 12$ against $H_1: \mu < 12$, and the test statistic *t* = -2.5, and $\alpha = .05$?

ANSWER:
-2.5 < $-t_{17,.05} = -1.740$, so we would reject H_0. The data indicates that the pens do not meet the design specifications.

66. What conclusion is appropriate for the hypotheses $H_0: \mu = 12$ against $H_1: \mu < 12$, and the test statistic *t* = -2, and $\alpha = .01$?

ANSWER:
-2.0 is not $< -t_{17,0.01} = -2.567$, so we would not reject H_0. There is not enough evidence to say that the pens don't satisfy the design specifications.

67. A company produces electrical devices operated by a thermostatic control. According to the engineering specifications, the standard deviation of the temperature at which these controls actually operate should not exceed 2.0 degrees Fahrenheit. For a random sample of 25 of these controls, the sample standard deviation of operating temperatures was 2.36 degrees Fahrenheit. Stating any assumptions you need to make, test at the 5% level the null hypothesis that the population standard deviation is at most 2.0 against the alternative that is bigger.

ANSWER:
We conclude that the population standard deviation is at most 2 degrees Fahrenheit.

68. Explain carefully the distinction between null and alternative hypotheses.

ANSWER:
The null hypothesis is the statement that is assumed to be true unless there is sufficient evidence to suggest that the null hypothesis can be rejected. The alternative hypothesis is the statement that will be accepted if there is sufficient evidence to reject the null hypothesis

<u>THE NEXT THREE QUESTIONS ARE BASED ON THE FOLLOWING INFORMATION:</u>
Before agreeing to purchase a large order of polyethylene sheaths for a particular type of high pressure oil-filled submarine power cable, a company wants to see conclusive evidence that the true standard deviation of sheath thickness is less than .05 inches.

69. What hypotheses should be tested, and why?

ANSWER:
Let σ denote the population standard deviation. The appropriate hypotheses are $H_0 : \sigma^2 = (.05)^2$ against $H_1 : \sigma^2 < (.05)^2$. With this formulation, the burden of proof is on the data to show that the requirement has been met (the sheaths will not be used unless H_0 can be rejected in favor of H_1.)

70. In this test, what would be a Type I error?

ANSWER:
The type I error would be concluding that the standard deviation is < .05 inches when it is really ≥ 0.05 inches.

71. In this test, what would be a Type II error?

ANSWER:
The type II error would be concluding that the standard deviation is ≥ .05 inches when it is really < 0.05 inches.